Lecture 13

- Images of CMB
- Review
  - → cosmological theory of perturbations
    chapter 2, 4, 5.1, 5.4, 5.5
    [chapter 9 → details]
- CMB anisotropies (continued), primordial spectrum (5.4)
- Inflation (homogeneous approximation)
  - → motivation (generation of Big-Bang initial conditions)
  - → Modes exiting horizon
  - → slow-roll inflation

$$\frac{\eta_0}{\eta_r} \sim 50$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

$$T^\mu{}_\nu = \bar{T}^\mu{}_\nu(\eta) + \delta T^\mu{}_\nu(x)$$

$$h_{\mu\nu}(\eta, x) = \int d^3k\, e^{ikx} h_{\mu\nu}(k)$$

scalars, vectors, tensors +

Newtonian – conformal gauge

$$ds^2 = a^2(\eta)\left(-(1+2\Phi)\,d\eta^2 + (1+2\Psi)\,dx^2\right)$$

$\Phi = -\Psi$ [From EOM]

$$\delta p_\lambda = c^2_{s,\lambda}\,\delta\rho_\lambda$$

leaves: $\delta\rho_\lambda$, $v_i \sim i k_i v$, Φ

We then got a closed-form equation for Φ:

$$\varphi'' + 3\frac{a'}{a}(1+c_s^2)\varphi' + c_s^2 k^2 \varphi = 0$$

Similar to the wave equation.

Superhorizon modes ~ constant

Subhorizon in RD:

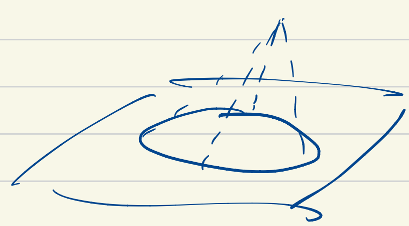
$$\Phi(\eta) = -3\Phi_{(i)} \cdot \frac{1}{(c_s k\eta)^2}\left[\cos(c_s k\eta) - \frac{\sin(c_s k\eta)}{c_s k\eta}\right] \underset{k\to\infty}{\sim} \frac{\cos c_s k\eta}{(c_s k\eta)^2}$$

(In MD ~ linear growth)

- Photons coming from different directions have a slightly different temperature:

$$\frac{\delta T}{T}(\theta) \sim \delta_{B\gamma}(\theta) + \qquad \theta \in S^2$$

$+ SW$



$6\ \varphi_i \cos u_s k \eta$

$u_s k_n \eta_r \sim \pi n \qquad k_n = \dfrac{\pi n}{u_s \eta_r}$

$$\Delta x = \frac{2\pi}{k_n} = \frac{2 u_s \eta_r}{n}$$

$$\Delta\theta = \frac{2 u_s \eta_r}{n \eta_0}$$

$$\frac{2\pi}{\Delta\theta} \simeq \ell_n \rightarrow \ell_n = \frac{2\pi \eta_0 n}{2 u_s \eta_r} = \pi\sqrt{3}\,\frac{\eta_0}{\eta_r}\, n \simeq 300\, n$$

more complicated, Legendre's

→ Gaussian random field

$$\langle \varphi_{k_1}\, \varphi_{k_2} \rangle = \delta(\vec{k_1} - \vec{k_2})\, P_{|k|}$$

→ Random but correlated (average over modes with same $|\vec{k}|$, different angles ~ ~ fixed ℓ → produces the plot, sensitive to $P_{|k|}$ (also Ω_m, Ω_B) and Ω_Λ, Ω_c

- We discussed evolution of initial conditions from Big Bang singularity until the period when they can be observed.

- Results of observations give

$$\delta \sim 10^{-5} \left(\frac{k_*}{k}\right)^{3+(n_s-1)}$$

→ perturbations are small and approximately gaussian

→ they are correlated on super-horizon scales

→ they have a relatively simple power-spectrum.

- This does not look like a generic explosion "Big Bang" and suggests to look for a simple theory that preceeds the Big Bang

- The most popular candidate is Inflation — a period of quasi-exponential expansion beyond the BB.
- There are numerous microscopic realizations of inflation that lead to similar observations
- We will discuss a particular model - single field slow-roll inflation first in the homogeneous approximation
- Then (next lecture) we will show that quantum mechanics creates classical perturbations that naturally have the required properties.

dS recap (lecture 2)

$w = -1$, or cosmological constant case, flat slicing:

$\rho = \text{const}$, $a = e^{Ht}$

$H = \sqrt{\frac{\Lambda}{3}}$ in our notation.

there is no singularity at $t = -\infty$ ($R = \text{const}$)

Global de Sitter can be obtained using closed slicing:

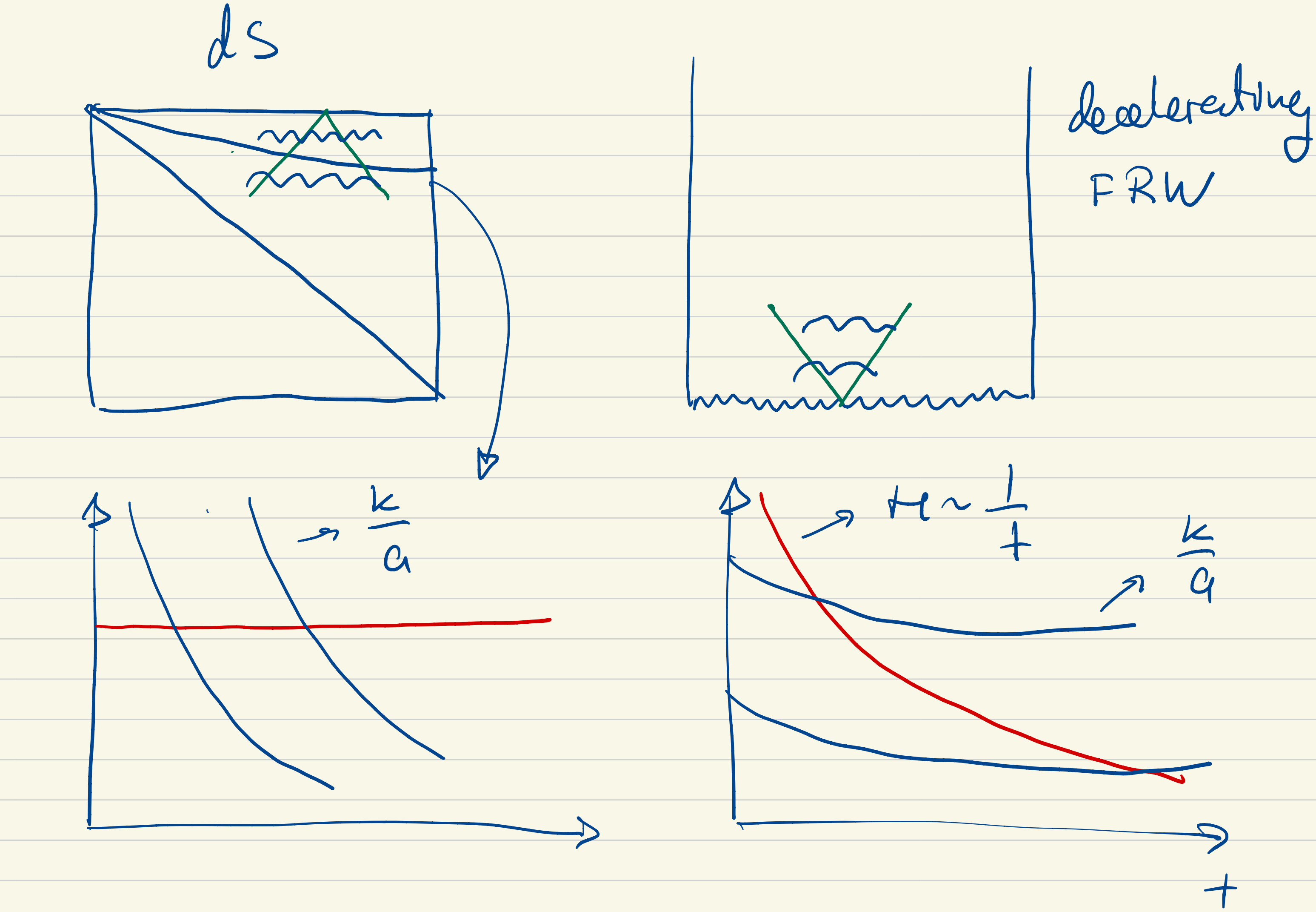
$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \frac{\Lambda}{3} = 0 \qquad a = H^{-1} \cosh Ht$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \Lambda = 0$$

$$\Downarrow$$

$$H^2 = \frac{\Lambda}{3} \quad \text{(both equations)}$$

We are interested in the expanding part of de Sitter space.



• Problem: de Sitter space, if produced by a rigid cosmological constant lasts forever - matter or radiation never dominates

$$H = H_0 \cdot \sqrt{\underline{\underline{\Omega_\Lambda}} + \Omega_M (1+z)^3 + \Omega_{rad} (1+z)^4}$$

• Solution:

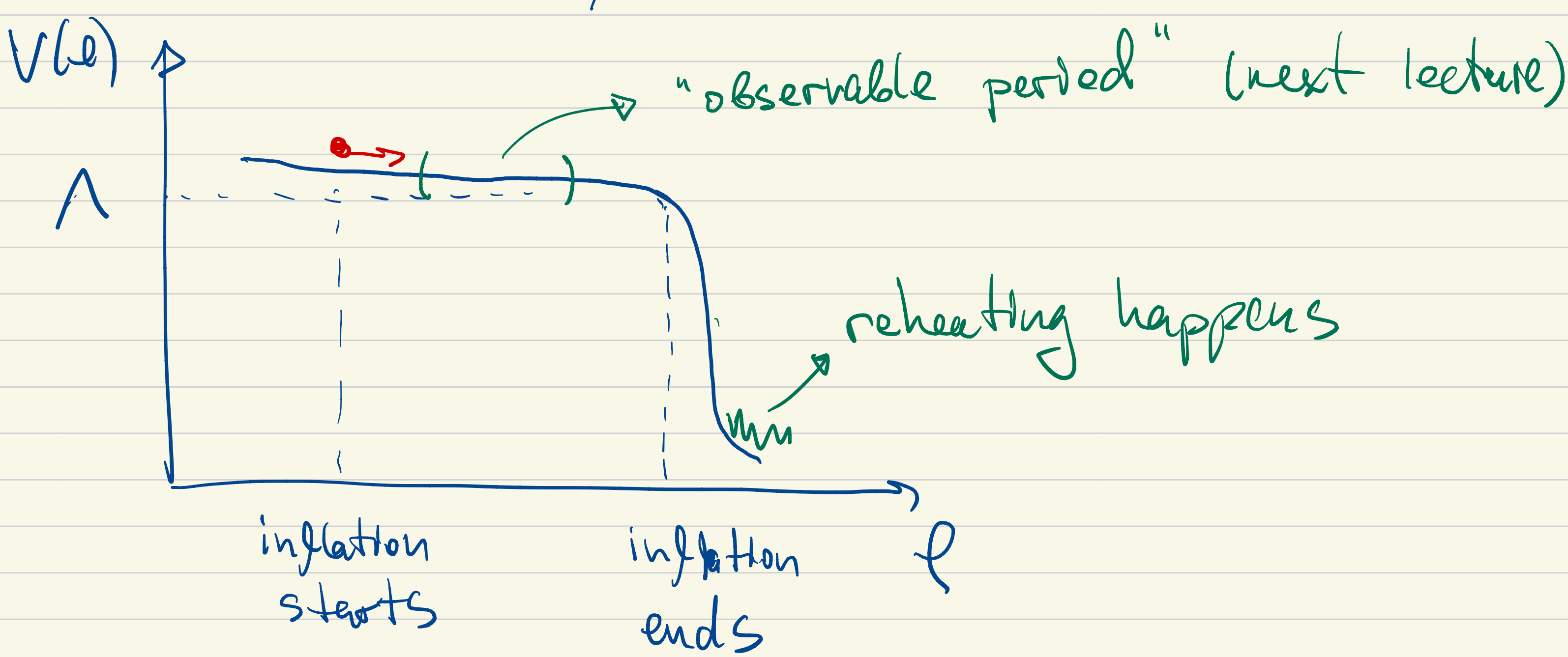
Inflation = Λ + "clock"

clock tells when Λ "decays"

[this leads to some t-dependence of Λ]

~ quasi de Sitter space

A concrete realisation: single-field slow-roll inflation



reheating: energy ~ Λ gets transmitted into radiation $T^4 \sim \Lambda$ and matter

Universe "heats up" [is scalar field a surprise?]

- $V(\varphi)$ does not need to have such a particular form, but good to imagine it for simplicity.

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right],$$

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = \partial_\mu \varphi \partial_\nu \varphi - g_{\mu\nu} \mathcal{L}$$

We first use an approximation $a = e^{Ht}$

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0$$

$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$$

$$p = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$$

$$H^2 = \frac{8\pi}{3M_{pl}^2}\left(\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right)$$

slow roll approximation:

$$\left|\frac{\ddot{\varphi}}{3H\dot{\varphi}}\right| \ll 1 \qquad (**)$$

$$H = \text{const} \rightarrow p \simeq -\rho \Rightarrow$$

$$\frac{\dot{\varphi}^2}{2V(\varphi)} \ll 1 \qquad (*)$$

Now EOM simplifies:

$$\dot{\varphi} \simeq -\frac{1}{3H}V' \qquad H = \frac{1}{M_{pl}}\left(\frac{8\pi V}{3}\right)^{1/2}$$

$$\Rightarrow \frac{\dot{H}}{H} = -\frac{3}{2}\frac{\dot{\varphi}^2}{V}H \quad + (*) \rightarrow \frac{\dot{H}}{H^2} \ll 1$$

[indeed dS]

Let us express the condition (*) purely in terms of the potential:

$$\frac{M_{pl}^2}{48\pi}\left(\frac{V'}{V}\right)^2 \ll 1 \qquad (1)$$

Now let us express condition (**),

$$\ddot{\varphi} \simeq M_{pl}\left(\frac{V''}{V^{1/2}} - \frac{1}{2}\frac{V'^2}{V^{3/2}}\right)\dot{\varphi} =$$

$$= M_{pl}^2\left(\frac{V''}{V} - \underbrace{\frac{1}{2}\left(\frac{V'}{V}\right)^2}_{(*)\ \to\ \text{small}}\right) H\dot{\varphi}$$

$$(**) \iff \frac{V''}{V} \ll \frac{24\pi}{M_{pl}^2} \qquad (2)$$

(1) and (2) are called slow-roll conditions on the potential.

$$\varepsilon = \frac{M_{pe}^2}{16\pi}\left(\frac{V'}{V}\right)^2 \qquad \eta = \frac{M_{pe}^2}{8\pi}\frac{V''}{V}$$

take $V = \frac{m^2}{2}\varphi^2$, conditions:

$$\frac{M_{pe}^2}{\varphi^2} \ll 1 \qquad \frac{M_{pe}^2}{\varphi^2} \ll 1$$

$$m^2\varphi^2 \ll M_{pe}^4$$